Problem Set 6
(Lecture 11 & Lecture 12)
Due: 10/16

Question numbers and pages based on 7th Edition, please make sure you check and do the proper questions if using a different version*

1. A random variable X has its pdf $f(x) = ce^{-\frac{x}{3}}$ for $x \in \mathbb{R}^+$, where $c(>0)$ is an appropriate constant.
    a. Determine c
    b. Find the $E\left[4\sqrt{\pi}X\right]$
2. Textbook Exercise 4.9
3. Textbook Exercise 4.19
4. Derive the MGF for $X \sim U(a, b)$
5. Consider the case where the waiting time X at a bus stop, measured in minutes, may be uniformly distributed between zero and five.
    a. What is the probability that someone at the bus stop would wait more than 3.8 minutes for the bus?
    b. What is the probability that someone at the bus stop would wait more than 5 minutes for the bus?
    c. What is the probability that someone at the bus stop would wait within 1 standard deviation of the mean wait time for the bus? (hint: round the standard deviation to 2 decimal places before finding the interval you are interested in).
6. Consider the random variable Y with a uniform distribution over the interval (0,1).
    a. Confirm that $P(a \leq Y \leq a + b)$, for $a \geq 0$, $b \geq 0$, and $a + b \leq 1$, depends only upon the value of b.
    b. Use the integration approach to find the $V[X]$ (hint: use the variance equality formula regarding expectation). You will not get any credit for using the shortcut formula for variance of a continuous uniform distribution.
    c. Can you think of a real-world application of something that would follow a continuous uniform distribution over the interval (0,1)?