

**Problem Set 9**  
**(Lecture 17 & Lecture 18)**  
**Due: 11/8**

**\*\*Question numbers and pages based on 7<sup>th</sup> Edition, please make sure you check and do the proper questions if using a different version\*\***

1. Textbook Exercise 5.25
2. Suppose that  $X_1$  and  $X_2$  have the joint pmf  $f(x_1, x_2) = p^2 q^{x_2}$ ,  $x_1 = 0, 1, 2, \dots, x_2$  and  $x_2 = 0, 1, 2, \dots$  with  $0 < p < 1$ ,  $q = 1 - p$ .
  - a. Find the marginal distributions of  $X_1$  and  $X_2$  (hint: consider the geometric series)
  - b. Find the condition distribution  $X_1 | X_2 = x_2$
3. Suppose that  $X_1$  and  $X_2$  have the joint pdf given by  $f(x_1, x_2) = \begin{cases} 1, & \text{if } 0 < x_1, x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$ .  
 Show that  $P\{X_1 X_2 > a\} = 1 - a + a \log(a)$  for any  $0 < a < 1$  {hint: note that  $P\{X_1 X_2 > a\} = \int_{x_1=a}^1 \int_{x_2=\frac{a}{x_1}}^1 f(x_1, x_2) dx_2 dx_1$ }.
4. Consider the joint distribution of  $X_1, X_2, X_3, X_4$   
 $f(x_1, x_2, x_3, x_4) = \lambda^4 e^{-\lambda(x_1 + x_2 + x_3 + x_4)}$  for  $0 < x_1, x_2, x_3, x_4 < \infty$ ,  $\lambda > 0$ 
  - a. Are the  $X_i$  for  $i = 1, 2, 3, 4$  independent?
  - b. What is the marginal distribution of  $X_3$ ,  $f_3(x_3)$ ? If it is a distribution that we have covered in class, properly label this distribution and identify the parameter(s).
5. Use the joint distribution from Question 4 to answer the following
  - a. Find  $E[X_1 X_2 X_3 X_4]$
  - b. Find  $E[3X_1 + 5(X_2)^3 - 6X_3 + (X_4)^2]$
6. Suppose that the random variables  $X_1$  and  $X_2$  have the following joint distribution

$X_2$ Values	$X_1$ Values		
	0	1	2
0	0	$\frac{3}{15}$	$\frac{3}{15}$
1	$\frac{2}{15}$	$\frac{6}{15}$	0
2	$\frac{1}{15}$	0	0

Evaluate  $E[X_1^2 X_2]$ ,  $E[X_1^2 X_2^2]$ ,  $E[X_1 X_2^3]$ ,  $E[X_1^2 X_2(1 - X_2)]$