

Problem Set 3
(Lecture 5 & Lecture 6)
Due: 9/18

*****Question numbers and pages based on 7th Edition, please make sure you check and do the proper questions if using a different version*****

1. In the 1970's show "Let's Make a Deal," a contestant was first given a choice of 3 doors to select the one they believed the prize lay behind (a goat was behind the 2 other). After the contestant chose which door they thought the prize was behind, the host would then reveal one of the other two doors that contained a goat instead of the prize. Finally, the host would ask if the contestant would like to switch doors to the other remaining door. The question is: Should the contestant switch? To answer this question, answer the following:
 - a. If you pick Door 1 (D1), what is your probability of winning the prize?
 - b. Now the host, who knows what's behind the doors, decided to give you more information. He will open another door that does not have the car behind it. He will not open Door 1 since you have picked it. What are the probabilities of the host opening Door 2 and Door 3? You may use O1, O2, or O3 to denote the events that he opens Door 1, Door 2, or Door 3, respectively.
 - c. Suppose that the host opens Door 3 (O3), which has a goat. He then gives you another opportunity to choose. Given this event, calculate the probabilities of winning the prize if you stick to your original choice, and if you switch to pick Door 2. Based on this, should you stick to your original choice or switch?
2. Polygraph tests (lie detector tests) are often routinely administered to employees or prospective employees in sensitive positions (or every time on Criminal Minds...). According to studies of polygraph reliability (Gastwirth, J., 1987. The Statistical precision of medical screening procedures, Statistical Science, 3, 213-222) if a person is lying, the probability that this is detected by the polygraph is .88, whereas if the person is telling the truth, the polygraph is indicating that he/she is telling the truth with probability .86. Now, suppose that on a particular question the vast majority of subjects have no reason to lie, so that 99% tell the truth.
 - a. Write down the 3 pieces of given information in terms of the probability of events A (the polygraph reading is positive – meaning the machine thinks the person is lying) and B (the subject is telling the truth).
 - b. Draw a probability tree for this problem.
 - c. What is the probability that the polygraph produces a positive reading?
 - d. Given that a subject produces a positive reading on the polygraph, what is the probability that the polygraph is incorrect and that he/she is telling the truth?
3. Use the law of total probability to prove that if $P(A|C) > P(B|C)$ and $P(A|C^C) > P(B|C^C)$, then $P(A) > P(B)$.
4. Textbook Exercise 2.135
5. Suppose you work for Austin's new company Menger Education, LLC (true story...). The company hosts classes training high school students in Microsoft office productions in

preparation for the Microsoft certifications they put out. Each student pays \$500 for the 2-month course, and Austin offers a guarantee that if the student doesn't pass the certification test, then he will refund the class and pay him/her back the \$100 test fee (this fee was paid to Microsoft, NOT Austin). Austin advertises that 91% of his students pass the exam.

- a. Let the random variable Y be the company's gain per student. Is Y a discrete or continuous random variable? What is the probability distribution function of Y in table form? (hint: there are only 2 possible values of Y).
 - b. What is the expectation with regard to company profit (gain) for the certification class for 1 student?
 - c. Suppose that in 2 years more and more students start to sign up for the certification classes. So, the passing rate for certifications will naturally decrease, say to .87. Naturally, this will have to be reflected in the price each student pays for the course. If Menger Education, LLC intends to keep the same expected profit (gain) per student, what should the cost of the course be per student? (hint: you may need to make a new table...)
6. Choose an adult age 26-30 at random, and let the random variable X be the number of significant others the individual has had in his/her lifetime so far.

x	0	1	2	3	4	5	6	7	8
P(x)	0.135	0.271	0.271	0.180	0.09	0.036	0.012	0.003	?

- a. First find the $P(x = 8)$
 - b. Austin's parents are concerned that he has recently broken up with his second girlfriend. Austin claims that this is not unusual. What is the probability that a randomly selected adult age 26-30 has as many or more significant others than Austin?
 - c. Given that by age 28, Austin has had two significant others already, what is the probability that he will have no more than four significant others in his life?
7. Consider a company with 2 production lines. The following probability distribution function of the random variables X and Y represents the daily number of defective products coming out of each production line.

x,y	0	1	2	3	4
P(x)	.15	.30	.25	a	.10
P(y)	.05	b	.10	.75	.05

- a. Find a and b
- b. Which production line, X or Y , would you intuitively expect to have the higher average daily number of defective products coming out of it? Answer this question only verbally, calculations will receive 0 credit.
- c. Which production line, X or Y , would you intuitively expect to have a higher standard deviation? Answer this question only verbally, calculations will receive 0 credit.
- d. Use the variance equality from Lecture 6 to confirm your answer to part (c).