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3375Q

Problem Set 11
(Lecture 21 & Lecture 22)
Due: 12/6

*****Question numbers and pages based on 7th Edition, please make sure you check and do the proper questions if using a different version*****

1. Textbook Exercise 6.63
2. Textbook Exercise 6.71
3. Suppose that (X_1, X_2) has the following joint pdf
$$f(x_1, x_2) = \begin{cases} 1, & 0 < x_1 < 2, 0 < x_2 < 1, 2x_2 \leq x_1 \\ 0, & elsewhere \end{cases}$$
Find the pdf of $U = X_1 - X_2$ (hint: use the transformation $u = x_1 - x_2$ and $v = x_1$).
4. Consider 3 independent and identically distributed (iid) continuous random variables X_1, X_2, X_3 each $U(0,4)$.
    a. What is the joint distribution $f_{1,2,3}(x_1, x_2, x_3)$
    b. What is the joint distribution $f_{2,3}(x_2, x_3)$?
    c. What is the marginal distribution of X_3?
    d. What is the distribution of $X_{(3)}$?
5. Having studied how to calculate the distributions of the smallest and largest order statistics, another interesting application is the joint distribution of just $(X_{(1)}, X_{(n)})$. Heuristically, since $x_{(1)}$ and $x_{(n)}$ are assumed fixed, each of the remaining $n - 2$ order statistics can be anywhere between $x_{(1)}$ and $x_{(n)}$, while these could be any $n - 2$ of the original n random $X's$. Now, $P\{x_{(1)} < X_i < x_{(n)}\} = F(x_{(n)}) - F(x_{(1)})$, for each $i = 1, 2, \ldots, n$. Hence, the joint pdf of $(X_{(1)}, X_{(n)})$ would be given by
$$f(x_{(1)}, x_{(n)}) = \begin{cases} \left(\frac{n!}{(n-2)!}\right)\{F(x_{(n)}) - F(x_{(1)})\}^{n-2} f(x_{(1)}) f(x_{(n)}) & -\infty < x_{(1)} < x_{(n)} < \infty \\ 0 & elsewhere \end{cases}$$
    a. Consider 3 independent and identically distributed (iid) continuous random variables X_1, X_2, X_3 each $U(0,1)$. Show that the joint pdf of $(X_{(1)}, X_{(3)})$ is
    $$f(x_{(1)}, x_{(3)}) = \begin{cases} 162(x^2 - x)^3 & 0 < x < 1 \\ 0 & elsewhere \end{cases}.$$
    b. What would be an actual practical example where we would be interested in the joint distribution of just the min and max order statistics?
6. Textbook Exercise 6.81