Honors Conversion

In order to achieve an honors conversion, you must complete the following:

1. Independent study of the following additional materials with guidance from the instructor is required:
   - Sections 3.9, 6.5, 6.6 of the textbook
   - 6 Problems from each of the above sections. For you this will be
     - Section 3.9: 3.151, 3.155, 3.157, 3.159, 3.161, 3.163,
     - Section 6.5: 6.41, 6.45, 6.47, 6.51, 6.53, 6.59
     - Section 6.6: 6.63, 6.65, 6.66, 6.67, 6.69, 6.71
2. 2 extra homework problems per week, from the same sections covered in the homework

For the independent study, Section 3.9 pertains to Lecture 10. Sections 6.5 and 6.6 can be covered after Lecture 21. Read the sections on your own and use the notes from class to try to solve the specified questions. I'd like those handed in with your Problem Set 11. Just remind me when you're handing them in.

Each problem set will have an extra 2 questions necessary each week for the honors conversion. These additional questions are listed below. Please hand them in with your submission for each respective problem set.

Also make sure you submit the honors conversion request (https://honors.uconn.edu/honors-course-conversions/) so that it can get approved.

Weekly Questions

*****Question numbers and pages based on 7th Edition, please make sure you check and do the proper questions if using a different version*****

Pset 1

- Consider the Sample space S = (0,1] and define the collection of sets $\{A_i; i \epsilon I\}$ where $A_i = \left(\frac{1}{2^i}, \frac{1}{2^{i-1}}\right), i \epsilon I = \{1,2,3, \dots\}$. Check that the given collection of intervals form a partition of (0,1].
- Compare and contrast stratified and cluster sampling. When is which option more appropriate?

Pset 2

- Consider a sample space S with events A,B, and C. Prove the following:
  - $P(A \cup B) \leq P(A) + P(B)$
  - $P(A \cap B) \geq P(A) + P(B) - 1$
  - $P(A \cap C) \leq \min\{P(A), P(C)\}$

- $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

- Suppose that A and B are 2 events, prove the following statements are equivalent:
  - The events A and B are independent
  - The events A^C and B are independent
  - The events A and B^C are independent
  - The events A^C and B^C are independent

Pset 3

- Suppose that a random variable X has the following pmf:

X values	-4	-2	1	3	6
Probabilities	$.3 - p^2$	p^2	$.1$	$2p$	$.4$

where $p^2 \leq .3$. Is it possible to determine p uniquely?

- Derive the formula for the variance of a discrete random variable.

Pset 4

- Suppose that a random variable X has the $Bin(n, p)$ distribution, $0 < p < 1$. Show that $P(X > 1 | X \geq 1) = \frac{1-(1-p)^n - np(1-p)^{n-1}}{1-(1-p)^n}$
- An urn contains 6 blue and four red marbles of identical size and weight. We reach in to draw a marble at random, and if it is red, we throw it back in the urn. Next, we reach in again to draw another marble at random, and if it is red then it is thrown back in the urn. Then, we reach in for the third draw and the process continues until we draw the first blue marble. Let X be the total number of required draws. What is the probability that we will need to draw marbles fewer than four times?

Pset 5

- Suppose that a random variable X has the $Poisson(\lambda)$ distribution, $0 < \lambda < \infty$. For all $x = 0,1,2, \ldots$, show that $\frac{P(X=x)}{P(X=x+1)} = \frac{(x+1)}{\lambda}$ (this recursive relation helps enormously in computing the Poisson probabilities successively for all x, particularly when x is large).
- A random variable X has its MGF given by $M_X(t) = \frac{1}{5}\left\{\frac{1+2e^{2t}}{e^{4t}} + \frac{e^{3t}+e^{6t}}{e^{5t}}\right\}$ for $t \in \mathbb{R}$. Find μ and σ^2. Can the distribution of X be identified?

Pset 6

- Consider the expression of the MGF of an exponential distribution $M_X(t) = (1 - \beta t)^{-1}$ for all $t < \beta^{-1}$
  - Confirm that the $V[X] = \beta^2$
  - Find the third and fourth moments of the exponential distribution
- Prove that if a random variable X has a finite MGF $M_X(t)$, for $|t| < a$ with some $a > 0$, then the r^{th} moment η_r of X is the same as $\frac{d^r M_X(t)}{dt^r}$ when evaluated at $t = 0$. Assume that X is a continuous random variable, and that the differentiation operator of $M_X(t)$

w.r.t. t can be taken inside the integral w.r.t x (i.e. $\frac{d}{dt}\int_X f(x)dx = \int_X \frac{d}{dt}[f(x)]dx$). (hint: there is no need to use formal induction, just show that it holds for η_1 and η_2 and then state that the pattern would continue). How would this proof change if the random variable X were discrete?

Pset 7

- Derive the generalized formula depending only on n for $\Gamma\left(n+\frac{1}{2}\right), n \in \mathbb{Z}^+$. Confirm that when you plug $n = 4$ into your formula you get $\frac{105}{16}\sqrt{\pi}$.
- Suppose $Y = 3Z$ where $Z \sim N(0,1)$, find $M_Y(t)$.

Pset 8

- Textbook Exercise 4.197
- Textbook Exercise 5.17

Pset 9

- Suppose that X_1 is distributed as $N(\mu, \sigma^2)$ and conditionally the distribution of X_2 given that $X_1 = x_1$ is $N(x_1, \sigma^2)$. Then, show that the joint distribution of (X_1, X_2) is given by $N_2\left(\mu, \mu, \sigma^2, 2\sigma^2, \frac{1}{\sqrt{2}}\right)$
- Suppose that X_1 and X_2 have the joint pmf such that $P\{X_1 = 2 \cap X_2 = 3\} = \frac{1}{3}$, $P\{X_1 = 2 \cap X_2 = -1\} = a$, $P\{X_1 = -1 \cap X_2 = 3\} = b$, and $P\{X_1 = -1 \cap X_2 = -1\} = \frac{1}{6}$ where a and b are appropriate numbers. Determine a and b when X_1 and X_2 are independent.

Pset 10

- With any 2 random variables X_1 and X_2, show that $Cov(X_1 + X_2, X_1 - X_2) = V[X_1] - V[X_2]$, provided that $Cov(X_1, X_2), V[X_1]$, and $V[X_2]$ are finite.
- Let the pdf of X be $f(x) = \frac{1}{\beta}e^{-\frac{x}{\beta}}$ for $x > 0$ with $\beta > 0$. Find the pdfs of the following random variables which are functions of X.
  - $U = X^2$
  - $V = X^3$

Pset 11

- Suppose that (X_1, X_2) is distributed $N_2(0,0,\sigma^2,\sigma^2,\rho)$ with $0 < \sigma < \infty, -1 < \rho < 1$. Define $U = \frac{X_1}{X_2}$ and $V = X_1$.
  - Find the joint pdf of U and V
  - Derive the marginal pdf of U by integrating out V
  - Show that when $\rho = 0$, the marginal pdf of U coincides with the standard Cauchy pdf, namely $\pi^{-1}(1+u^2)^{-1}$ for $-\infty < u < \infty$.
- Textbook Exercise 6.85