Austin Menger
3375Q

Problem Set 7
(Lecture 13 & Lecture 14)
Due: 10/23

*****Question numbers and pages based on 7th Edition, please make sure you check and do the proper questions if using a different version*****

1. By the appropriate substitutions, express the following in the form of a gamma integral. Then evaluate these integrals in terms of the gamma functions.
   a. $\int_0^\infty xe^{-\frac{1}{3}x}dx$
   b. $\int_0^\infty x^2e^{-\frac{1}{3}x}dx$
   c. $\int_0^\infty xe^{-\frac{1}{3}x^2}dx$
2. There was a notion originally put out by Abraham de Moivre and eventually critiqued by James Stirling regarding approximations for factorials. Stirling's Approximation, as it has come to be known, states that $n! \sim \sqrt{2\pi}e^{-n}n^{n+\frac{1}{2}}$ for large values of n (it actually turns out that this approximation for $n!$ works well even for n as small as five or six). Stirling's approximation can actually be applied to estimate $\Gamma(\alpha)$.
   a. Which value of α would allow such an application?
   b. What would the final approximation for $\Gamma(\alpha)$ be?
3. Assuming $X \sim N(\mu, \sigma^2)$, derive $M_X(t)$.
4. Derive the variance of the Normal distribution assuming we know that $E[X] = \mu$.
5. Suppose that $X \sim N(\mu, \sigma^2)$, find η_3, the third moment.
6. Now, suppose that $Z \sim N(0,1)$, the standard normal distribution. What is η_1? What is η_3? How about η_5? Notice anything?