

**Problem Set 10**  
**(Lecture 19 & Lecture 20)**  
**Due: 11/29**

**\*\*Question numbers and pages based on 7<sup>th</sup> Edition, please make sure you check and do the proper questions if using a different version\*\***

1. Suppose that  $X_1$  and  $X_2$  have the joint pdf given by

$$f_{1,2}(x_1, x_2) = \begin{cases} 3(x_1^2 x_2 + x_1 x_2^2) & 0 < x_1, x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Show that  $\rho_{X_1, X_2} = -\frac{5}{139}$ . In the context of a multiple linear regression model that is modeling  $Y = \text{height}$  based on  $X_1 = \text{weight}$  and  $X_2 = \text{intelligence}$ , does  $\rho_{X_1, X_2}$  indicate that an interaction term should be included?

2. Consider the following marginal distributions for  $x_1, x_2, x_3$

$$f_1(x_1) = \begin{cases} 2x_1 & 0 < x_1 < 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$f_2(x_2) = \begin{cases} \frac{3}{8}x_2^2 & 0 < x_2 < 2 \\ 0 & \text{elsewhere} \end{cases}$$

$$f_3(x_3) = \begin{cases} x_3 + 2x_3^3 & 0 < x_3 < 1 \\ 0 & \text{elsewhere} \end{cases}$$

and assume  $X_1, X_2, X_3$  are independent.

- a. Find the joint distribution  $f_{1,2,3}(x_1, x_2, x_3)$  and confirm this joint distribution is a valid pdf.
  - b. Show that the fact that  $x_2$  and  $x_3$  already occurred does not impact the likelihood of  $x_1$  occurring, i.e.  $f_{1|2,3}(x_1) = f_1(x_1)$
  - c. Find  $E[X_1|X_2 = x_2, X_3 = x_3]$  and  $E[X_1 X_3^2|X_2 = x_2]$  with  $0 < x_2 < 2, 0 < x_3 < 1$ .
3. Suppose that the random variables  $X_1$  and  $X_2$  have the following joint distribution

$X_2$ Values	$X_1$ Values		
	0	1	2
0	0	$\frac{3}{15}$	$\frac{3}{15}$
1	$\frac{2}{15}$	$\frac{6}{15}$	0
2	$\frac{1}{15}$	0	0

$$\text{Find } E \left[ \left( \frac{X_1^2 \sqrt{1+X_1}}{3} \right)^3 \middle| X_2 = 1 \right].$$

4. Textbook Exercise 6.5
5. Textbook Exercise 6.31
6. Textbook Exercise 6.41